RESONANCE GAS OSCILLATIONS IN A TUBE WITH AN OPEN END UNDER WEAKLY DEVELOPED TURBULENCE

R. G. Galiullin, É. R. Galiullina, and
E. I. Permyakov

Resonance gas oscillations in a tube, one end of which has an oscillating piston and the other of which is open to the environment, are considered. A semi-empirical model of resonance oscillations in a long tube under conditions of weakly developed turbulence is constructed. The boundary condition at the open end is calculated analytically. The relations obtained are compared to experiment.

Resonance oscillations are known to develop in a tube at one end of which a harmonically oscillating piston is placed and whose other end is open to the environment. The oscillations are accompanied by the formation of an oscillating jet at the open end, by flow turbulization, and by other nonlinear effects [1-4]. The interest in such systems is maintained due to their widespread use in technology.

The development of a quantitative theory of the phenomenon is restricted by the complexity of the boundary condition at the open end [5-7] and the poor investigation of oscillating turbulent flows inside a tube [8]. Recently, we managed to show [9] that the boundary condition can be found analytically if the concept of the jet character of efflux and spherical inflow into a tube is used [10]. To model intertube turbulence an approach is suggested [11] that is based on linearization of turbulent shear stress on the wall. One of the first models [12] includes the presence of an empirical coefficient in the boundary condition and assumptions of quasistationarity of an oscillating flow and the absence of heat exchange between the gas in a tube and the wall.

In what follows, we attempt to construct a model of resonance oscillations under turbulent conditions that is free of the constraints mentioned.

Oscillations in a long cylindrical tube ($L/R \gg 1$) that are excited by the harmonic motion of a piston with amplitude $l_0 \ll L$ are characterized by dimensionless parameters [7, 8, 10, 12, 13]:

$$\varepsilon = \frac{V}{\omega L}, \quad H = R \sqrt{\left(\frac{\omega}{\nu}\right)}, \quad \text{Sh} = \frac{\omega R}{V}, \quad M_p = \frac{\omega L_0}{c_0}, \quad \text{Re}_\omega = \frac{V^2}{\omega \nu}. \tag{1}$$

The condition $l_0 \ll L$ with oscillation on the base frequency provides $M_p \ll 1$. In the case most interesting for practice, $H \gg 1$, $\text{Sh} \leq 1$. The latter condition for $L/R \gg 1$ results in $\varepsilon \ll 1$, i.e., the problem can be solved by the perturbation method. Let turbulence be weakly developed. Then the universal profile of velocity fluctuations appears only in a thin layer near the wall [8].

Nonisoentropic gas oscillations in the tube can be described by the system [14]

$$\frac{\partial (\rho u)_s}{\partial t} + \frac{\partial (\rho u^2)_s}{\partial x} + \frac{\partial p}{\partial x} + \frac{2\tau}{R} = 0,$$

$$\frac{\partial p}{\partial t} + u_s \frac{\partial p}{\partial x} + \kappa \rho \frac{\partial u_s}{\partial x} - \frac{2(\kappa - 1) q}{R} = 0. \tag{2}$$

Equations of the first (acoustic) approximations are presented as

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$$\rho_0 \frac{\partial u_{1s}}{\partial t} + \frac{\partial p_1}{\partial x} = -\frac{2\tau_1}{R} \, ,$$

$$\frac{\partial p_1}{\partial t} + \rho_0 c_0^2 \frac{\partial u_{1s}}{\partial x} = \frac{2(\kappa - 1) q_1}{R} \, . \tag{3}$$

The relation between the amplitude of shear stress on the wall and maximum amplitude of velocity fluctuations has the form [8, 15]

$$\tau_{1m}^0 = \rho_0 f_w (u_{1m}^0)^2 / 2 \, . \tag{4}$$

In the case of weakly developed turbulence, $f_w \cong 0.005$ [8]. With resonance oscillations u_{1m}^0 is a function of the axial coordinate; therefore, expression (4) should be linearized. We use the method involving replacement of the parabola section $(u_{1m}^0)^2$ by the straight-line section $\beta_0 u_{1m}^0$ with the condition that the areas under both lines are equal [11]. Then we have

$$\tau_{1m}^0 = \rho_0 \beta_0 u_{1m}^0 \, , \quad \beta_0 = f_w V / 3 \, . \tag{5}$$

In (5), the fact that the limiting value of u_{1m}^0 in the open-end tube is attained at the open end, where $u_{1m}^0(L) = V$, is taken into account.

It may be taken as established [8, 15] that under the conditions of weakly developed turbulence the profile of the amplitude of velocity fluctuations is uniform everywhere except for the layer of thickness δ_1, where the distribution of the amplitude of velocity fluctuations is universal, i.e.,

$$u_1^+ = 2.5 \ln y^+ + 5.0 \, . \tag{6}$$

We take the relation between oscillations of shear stress on the wall and velocity fluctuations to have the form

$$\tau_1 = \rho_0 \beta u_{1s} \, , \quad \beta = \beta_0 / B \, . \tag{7}$$

Strictly speaking, there is a phase difference Φ^* between τ_1 and u_{1s} that can be allowed for by replacing β by $\beta^* = \beta \exp(i\Phi^*)$.

To determine q_1 we assume that

$$q_1 = -\beta_T p_1 \tag{8}$$

and introduce the thicknesses of dynamic acoustic and oscillating temperature boundary layers

$$\delta_1 = \sqrt{2\mu_e / \rho_0 \omega} \, , \quad \delta_{T1} = \sqrt{2\lambda_e / \rho_0 c_p \omega} \, , \tag{9}$$

In the adopted model, $\delta_1 \ll R$, $\delta_{th1} \ll R$; therefore, in the expressions

$$q_1 = \lambda_e \left. \frac{\partial T_1}{\partial r} \right|_w \, , \quad \tau_1 = -\mu_e \left. \frac{\partial u_1}{\partial r} \right|_w \tag{10}$$

the derivatives can be replaced by increment ratios. Then, with allowance for (7), (8), (10), and the condition that on the tube wall $u_1 = 0$, $T_1 = 0$, we can write

$$\lambda_e T_{1m} / \delta_{th1} \cong \beta_{th} p_1 \, , \quad \mu_e / \delta \cong \rho_0 \beta \, , \tag{11}$$

It can be easily shown that beyond the boundary-layer limits $p_1 \cong \rho_0 c_p T_{1m}$. Then it follows from (11) that

$$\beta_{th} = \beta / \sqrt{Pr_t} \, . \tag{12}$$

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Let (3) be written in a dimensionless form assuming $\bar{p}_1 = p_1/\rho_0 c_0^2$, $\bar{u}_{1s} = u_{1s}/c_0$. Then, with allowance for (7), (8), (12), we have

$$\frac{1}{c_0}\frac{\partial \bar{u}_{1s}}{\partial t} + \frac{\partial \bar{p}_1}{\partial x} = -a\bar{u}_{1s}\,, \quad \frac{1}{c_0}\frac{\partial \bar{p}_1}{\partial t} + \frac{\partial \bar{u}_{1s}}{\partial x} = -\frac{(\kappa - 1)}{\sqrt{\mathrm{Pr}_t}}\,a\bar{p}_1\,, \quad a = \frac{2}{3}\frac{f_w V}{BRc_0}\,. \tag{13}$$

The solutions of system (13) have the form

$$\bar{p}_1 = r_1 \cos(kx + \alpha_1 + i\beta_1)\exp(i(\omega t + \psi))\,,$$

$$\bar{u}_{1s} = -ir_1 \mu_1 \sin(kx + \alpha_1 + i\beta_1)\exp(i(\omega t + \psi + \varphi_1))\,, \tag{14}$$

where r_1, α_1, β_1, ψ are integration constants; μ_1, φ_1 are the module and the main value of the argument of the expression $[k/(k_0 - ia)]$, with

$$k^2 = k_0^2 - iak_0\left(1 + \frac{(\kappa - 1)}{\mathrm{Pr}_t}\right) - a^2\frac{(\kappa - 1)}{\mathrm{Pr}_t}\,. \tag{15}$$

Under the conditions of [7], $a \leq 0.05k_0$; therefore, with high accuracy

$$k \cong k_0 - ib\,, \quad b = (a/2)\left(1 + \frac{(\kappa - 1)}{\mathrm{Pr}_t}\right)\,. \tag{15'}$$

We formulate the boundary conditions. At the end closed by the piston, the piston velocity should be assigned, which in dimensionless form is [13]

$$\bar{u}_{1s}(0, t) = -i\,M_p \exp(i\omega t)\,. \tag{16}$$

We consider an oscillatory process near the open end of the tube provided with an infinite flange. Let at some distance from the outlet section (AA' in Fig. 1) the particle velocity be

$$u = V\cos\omega t\,. \tag{17}$$

We take the model of [10] assuming jet efflux ($u \geq 0$) and spherical inflow ($u \leq 0$) into the sink positioned at point O. The gas flowing from the tube is confined within the volume of the body with generatrixes BE and $B'E'$. Since the mixing layer does not succeed in substantially developing near the open end, the jet cross-section area remains practically constant and equals the tube cross-section area S_0. Then the gas velocity is also independent of x.

Inflow to the tube is achieved through hemispheres BB' and CC'. It can be easily shown [16] that the effect of viscous losses in suction is inessential. Then we can assume that gas inflow is potential and the hemispheres are isotachs. The quantity of gas crossing the hemispheres is constant; therefore, for the hemisphere of an arbitrary radius x, the relation

$$u(x, t) = \Phi(x)\,u^*(t)\,, \quad \Phi(x) = R^2/x^2 \tag{18}$$

is valid, where $*$ refers to hemisphere BB' drawn through the tube edges ($x = R$).

Consider the efflux through section DD' and the suction through hemisphere BB'. By virtue of the mass conservation law, the quantity of ejected gas should be compensated for by its return through BB', i.e.,

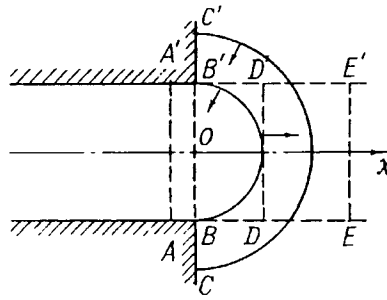$$S_0 \int_0^{t'} u_{1s}(t)\,dt + S\int_{t'}^T u^*(t)\,dt = 0\,, \tag{19}$$

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Fig. 1. Schematic of flow.

where $S = 2\pi R^2$. Since $S > S_0$, for (19) to be valid the duration of the efflux t' should be more than the duration of the suction. This is possible if the velocity has a constant increment. Assuming the latter to be proportional to the amplitude of velocity fluctuations V, we have

$$u = V(m_0 + \cos \omega t), \quad x = R, \tag{20}$$

where m_0 is an unknown parameter.

We find the efflux duration t' from the condition $u = 0$. Then we have

$$u_{1s}(\bar{t}) = BV(m_0 + \cos \bar{t}), \quad -(0.5\pi + \varphi) \leq \bar{t} \leq (0.5\pi + \varphi);$$

$$u^*(\bar{t}) = V(m_0 + \cos \bar{t}), \quad (0.5\pi + \varphi) \leq \bar{t} \leq (0.5\pi - \varphi), \tag{21}$$

where $\varphi = \arcsin m_0$; $B = u_{1s}^0 / V$ is the parameter allowing for flow displacement by the boundary layer. Substituting (21) into (19) can easily give

$$(B + 2) m_0 \pi + 2(B - 2)(m_0 \arcsin m_0 + \sin \arccos m_0) = 0. \tag{22}$$

To determine B it is enough to average the amplitude of velocity fluctuations over the tube cross section, i.e., to integrate the uniform profile with amplitude V from 0 to $(R - \delta_1)$ and profile (6) from $(R - \delta_1)$ to R.

With weakly developed turbulence $(10^5 < \text{Re}_\omega < 6 \cdot 10^5)$, the thickness δ_1 is found by the formula

$$\delta_1 R \cong 0.0154 V/\omega R. \tag{23}$$

As a result we have

$$B = 1 - (c \ln \text{Re}_\omega + d)(V/\omega R), \tag{24}$$

where $c = -0.00385$; $d = 0.0546$. Calculation shows that in the experiments of [7], corresponding to the problem at hand, the equality $B \cong 0.93$ is fulfilled with an accuracy to 1%. To this value of B corresponds $m_0 = 0.239$.

We consider oscillations of particles crossing, e.g., cross section EE' (Fig. 1), assuming the flow to be potential [17]. In efflux the velocity is determined by (20) and in suction by (18), according to which the velocity decreases rapidly with the growth of x. Expanding the velocity into Fourier series, we have

$$\bar{u} = M_E \left\{ (0.5m_0 + a_0) + (0.5m_0 - a_0) \Phi(x) + [(0.5 + a_1) + \right.$$

$$\left. + (0.5 - a_1) \Phi(x)] \cos \bar{t} + a_2(1 - \Phi(x)) \cos 2\bar{t} + ... \right\}, \tag{25}$$

$$a_0 = (1/\pi)(m_0\varphi + \cos \varphi), \quad a_1 = (1/\pi)(\varphi + 2m_0 \cos \varphi - 0.5 \sin 2\varphi),$$

$$a_2 = (1/\pi)[\cos \varphi - m_0 \sin 2\varphi - (1/3) \cos 3\varphi], \quad M_E = V/c_0, \quad \bar{u} = u/c_0.$$

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In the case of a tube without a flange, the function $\Phi(x)$ is determined by the formula

$$\Phi(x) = R^2 / [x^2 + (x - R)^2 + 0.5\pi R (x - R)].\tag{26}$$

An analysis shows that substantial changes in $\bar{u}_i$ are faster for the flange-free tube and stop at a distance $x \cong 3R$. Starting from $x \cong 5R$, the composition of oscillations stops depending on x and on the geometry of the open end. Then we have

$$\bar{u}_\infty = M_E [(0.5m_0 + a_0) + (0.5 + a_1) \cos \bar{t} + a_2 \cos 2\bar{t} + ...].\tag{27}$$

The flow in which particle velocity is determined by (25), (27) is an oscillating jet. The Lagrange–Cauchy integral can be applied to it:

$$\frac{p}{\rho_0} + \frac{u^2}{2} + \frac{\partial \varphi^*}{\partial t} = \text{const},\tag{28}$$

where, according to the estimate, the third term has the order of Sh. Applying (28) to two cross sections (e.g., AA' and EE', Fig. 1) under the assumption that the pressure in section EE' is equal to atmospheric pressure and the velocity is determined by (17) and (27), respectively, we have, after simple transformations,

$$\bar{p}_1 (L, t) = m \bar{u}_{1s}^0 \bar{u}_{1s} (L, t), \quad m = m_1 / B^2,\tag{29}$$

where $m_1 = (0.5 + a_1)(0.5m_0 + a_0 + 0.5a_2)$. Under the considered conditions with $B \cong 0.93$, $m_0 = 0.239$, we have $m_1 = 0.361$.

We place solutions (14) into (16) and (29); then to determine r_1, ψ, α_1, and β_1 we obtain the system of equations

$$r_1 \sin \alpha_1 \cosh \beta_1 = M_p \cos (\psi + \varphi_1), \quad r_1 \cos \alpha_1 \cosh \beta_1 = - M_p \sin (\psi + \varphi_1),$$

$$\cos z \cosh w = m \mu_1^2 r_1 \sqrt{\sin^2 z + \sinh^2 w} (\cos z \sinh w \cos \varphi_1 + \sin z \cosh w \sin \psi),$$

$$\sin z \sinh w = m \mu_1^2 r_1 \sqrt{\sin^2 z + \sinh^2 w} (\sin z \cosh w \cos \varphi_1 - \cos z \sinh w \sin \psi),$$

$$z = k_0 L + \alpha_1, \quad w = \beta_1 - bL,\tag{30}$$

which for $\sin \varphi_1 \sim \varepsilon^0$, $\sinh w \sim \varepsilon^0$, $r_1 \sim \varepsilon^0$ ($\varepsilon^0 \ll 1$), $\cos \varphi_1 \cong 1$, $\cosh w \cong 1$, $\mu_1 \cong 1$ admits the solutions

$$\alpha_1 = \pi/2 - kL, \quad \beta_1 = a_1 L + mr_1,\tag{31}$$

the amplitude r_1 is found from the equation

$$r_1 [\cos^2 k_0 L + (bL + mr_1)^2 \sin^2 k_0 L]^{1/2} = M_p.\tag{32}$$

By virtue of (13), (15'), we have $bL = Cr_1 / B^2$, where $C = (1/3)f_w[1 + (\kappa - 1)/\sqrt{Pr_t}]$; then with an accurate resonance $k_0 L \cong \pi/2$ for dimensionless amplitude of oscillations we obtain

$$r_1 = B \sqrt{\left(\frac{M_p}{CL/R + m_1}\right)}.\tag{33}$$

In the boundary layer of stationary turbulent flows, $Pr_t \cong 0.9$ [18]. Assuming this value of Pr_t acceptable for the case of oscillating flows as well, we find $C = 0.002369$.
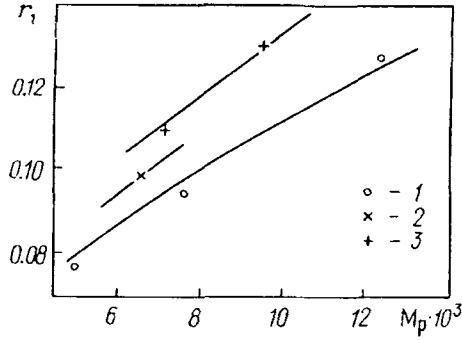
Fig. 2. Dependence of dimensionless amplitude of oscillations r_1 on M_p: points
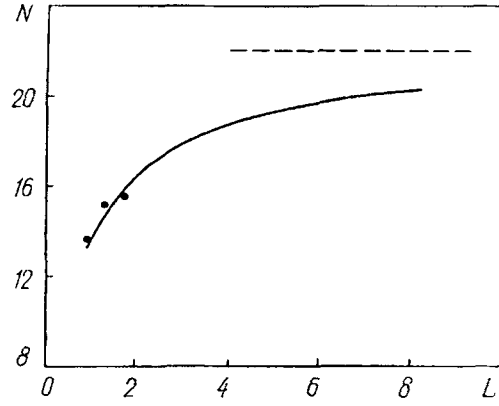— experiment [7] [1) $L/R = 171$; 2) 129; 3) 89]; curves — theory (33).

Fig. 3. Dependence of the coefficient of amplification on the tube length (m):
points — experiment [7]; solid curve — calculation by (34) at $l_0 =$
$5.53 \cdot 10^{-3}$ m; dashed line — experiment [19]. L, m.

The points in Fig. 2 indicate the experimental data of [7], relating to the conditions of weakly developed turbulence, in the form of $r_1 = r_1(M_p)$ and the results of calculations by (33). It is seen that the coincidence of the data is rather satisfactory: the deviation of points from calculations does not exceed 4%. Scatter of data is caused, as follows from (33), by the change in L/R.

Resonance properties are well manifested in comparing the amplitude of velocity fluctuations V at the open end with the amplitude of piston velocity fluctuations [19]. We formulate the relation $N = V/\bar{u}_{1s}^0$ (0) with allowance for $V = r_1 c_0/B$ and $\bar{u}_{1s}^0 = \pi l_0 c_0/2L$. Then we have

$$N = \sqrt{\left(\frac{2}{\pi l_0} \frac{L}{m_1 + CL/R} \right)}. \tag{34}$$

In Fig. 3, the points show experimental results of [7], and the solid curve indicates the results of calculation by (34). It is seen that as L increases, the value of N asymptotically tends to a constant value dependent on the tube radius (at $l_0 \cong$ const). A similar phenomenon was observed in [19] (the dashed curve in Fig. 3); the numerical values of N in [19] turn out to be 10% higher than those calculated.

Thus, an analytical solution of the problem of resonance oscillations in a tube with an open end under conditions of weakly developed turbulence has been obtained for the first time.

NOTATION

l_0, amplitude of piston motion; L, tube length; R, radius; c_0, sound velocity in unperturbed gas; μ, dynamic viscosity; ν, kinematic viscosity; λ, thermal conductivity; ω, cyclic frequency of oscillations; Sh, Strouhal number; M, Mach number; t, time; T, period of oscillations on frequency ω; t', efflux duration; x, axial coordinate; y, distance from the tube wall; δ_1, thickness of the acoustic dynamic boundary layer; δ_{th1}, thickness of oscillating temperature boundary layer; V, maximum amplitude of velocity fluctuations at the open end; p, pressure; u, axial velocity; $\bar{u}_\infty$, velocity at a distance from the open end; ρ, density; T_1, temperature fluctuations; $\kappa = c_p/c_v$; c_p, c_v, specific heats; k, wave number; $k_0 = \omega/c_0$; τ, shear stress on the wall; q, heat flux density on the wall; $y^+ = v^* y/\nu$; $u_1^+ = u_1^0/v^*$; v^*, shear stress rate; f_w, friction coefficient; $\text{Pr}_t = c_p \mu_e/\lambda_e$, turbulent Prandtl number; $\lambda_e = \lambda + \lambda_t$, $\mu_e = \mu + \mu_t$; $B = u_{1s}^0/u_{1m}^0$; $B_L = u_{1s}^0(L)/V$; φ^*, velocity potential. Subscripts: 0, unperturbed flow; 1, oscillations at frequency ω; 2, oscillations at frequency 2ω; s, averaging over the tube cross section; m, maximum value in the given cross section; e, effective (total) value; t, turbulent value; th, thermal; w, value on the wall; p, piston; E, closed end. Superscripts: 0, corresponds to the amplitude; bar denotes dimensionalization.

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